Speculative Bubbles in Present-Value Models: A Bayesian Markov-Switching State Space Approach

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Abstract

We incorporate a speculative bubble subject to a surviving and a collapsing regime into the present-value model by Binsbergen et al. (2010), who pioneer the latent variables approach to estimate expected returns and expected dividend growth rates. To estimate this new high-dimensional model, we develop an efficient Markov chain Monte Carlo sampler to simulate from the joint posterior distribution. We apply our present-value model to artificial as well as real-world datasets. Our setup is able to correctly identify 92.27% of all the bubble collapsing dates in the artificial datasets. And it never signals a bubble when there is none in the data generating process. We then show the existence of significant Markov-switching structures in real-world stock price bubbles. The results indicate that dividend growth rates are highly predictable. Further, we argue that present-value models should not ignore the bubble component of stock prices. Indeed, we find that bubble variation accounts for most of the variation in the price-dividend ratio in the US, UK, Malaysia and Japan, and more than 35% of the price-dividend variation in Brazil. Moreover, bubble variation explains also a large share of unexpected return variation.

Keywords: Periodically collapsing bubbles, Present-value approach, Return predictability, Markov-switching model, State space model, Bayesian analysis.

JEL classification: C11; C32; G12; G14.
1 Introduction

Variation through time in the price-dividend ratio on corporate stocks conveys essential information about expected returns or expected dividend growth rates (Campbell and Shiller, 1988). Recently, Binsbergen et al. (2010) have followed the insight of Cochrane (2008) to study jointly return and dividend dynamics in order to capture the information of the present-value relations between price-dividend ratios, expected returns and expected dividend growth rates. In particular, Binsbergen et al. (2010) pioneer a latent variables approach to estimate the expected returns and expected dividend growth rates of the aggregate stock market. They find that returns and dividend growth rates are predictable with $R^2$ values ranging from 8.2% to 8.9% for returns and 13.9% to 31.6% for dividend growth rates. More recently, Choi et al. (2017) show that incorporating regime shifts in the mean of price-dividend ratios into the present value model of Binsbergen et al. (2010) increases in-sample predictability. Another extension of Binsbergen et al. (2010) is contained in Piatti and Trojani (2017) which use a latent variables approach to estimate a present-value model with time-varying risks.

Despite the relevance of the phenomenon of periodically collapsing bubbles in stock prices, present-value approaches above do not account for it. According to the Efficient Market Hypothesis, stock prices should ‘fully reflect’ all relevant information (e.g., future dividend prospects) (Fama, 1970). Hence, any persistent deviation of share prices from fundamentals maybe interpreted as a sign of market irrationality. However, the price may well reflect a self-confirming belief that a variable or a group of variables, not related to fundamentals, influences prices. In this context, an explosive behaviour of stock prices may still be consistent with rational behaviour of economic agents, which is often referred to as a rational bubble in stock prices (Diba and Grossman, 1988).

This paper proposes to incorporate a speculative bubble subject to a surviving and a collapsing regime into the present-value model by Binsbergen et al. (2010). Specifically, we contribute to the literature on the present-value model in the spirit of Campbell and Shiller (1988) by allowing prices to deviate from fundamentals because of a latent rational bubble component subject to a surviving and a collapsing regime as in Al-Anaswah and Wilfling (2011) and Lammerding et al. (2013). Our framework allows us to estimate expected returns and expected dividend growth rates, as well as to identify bubble’s collapse dates.

In order to estimate this new high-dimensional model, we adopt the Bayesian approach and use Markov chain Monte Carlo (MCMC) methods to simulate from the joint posterior distribution. Indeed, when the number of model parameters is large, standard maximum likelihood estimation tends to be numerically unstable and may result in estimates that
are locally, but not globally, maximal. In contrast, MCMC methods are numerically more robust and can handle a large number of parameters and latent variables. In addition, one novel feature of our implementation is that it builds upon the band and sparse matrix algorithms for state space models developed in Chan and Jeliazkov (2009), McCausland et al. (2011) and Chan (2013), which are shown to be more efficient than the conventional Kalman filter-based algorithms.

The current paper is also closely related to the literature on the identification of speculative bubbles. Empirical papers have mainly proposed two different approaches for the detection of bubbles: indirect and direct bubble tests.

The first group of studies is based on the so-called indirect bubble tests. Here, the authors apply sophisticated cointegration and unit-root tests to a dividend-price relationship (see, e.g., Diba and Grossman, 1988; Evans, 1991; Froot and Obstfeld, 1991; Hall et al., 1999; Balke and Wohar, 2002; Sarno and Taylor, 2003; Bohl, 2003; Bohl and Siklos, 2004; Kanas, 2005; Jiang and Lee, 2007; McMillan, 2007; Cerqueti and Costantino, 2011; Phillips et al., 2011; Chen et al., 2016). Among the indirect tests to detect bubbles, some researchers have proposed a Bayesian approach; among them Li and Xue (2009); Miao et al. (2012); Check (2014); Shi and Song (2011); Fulop and Yu (2017).

The second group of studies, which are more relevant to this work, implements direct tests for speculative bubbles by explicitly formulating the existence of a bubble in the alternative hypothesis (see West, 1987; Wu, 1997; Al-Anaswah and Wilfling, 2011; Lammerding et al., 2013). The basic idea in the seminal paper of West (1987) is to compare two alternative estimators for one particular parameter. Specifically, West (1987) constructs the indirect estimators by regressing the stock price on a suitable set of lagged dividends, and by estimating the observable no-bubble Euler equation. Under the null hypothesis of no-bubble, the two sets of estimates should be the same, apart from sampling error. However, this equality of the two sets will not hold under the alternative hypothesis suggested by Blanchard and Watson (1982) of speculative bubbles. West (1987) finds that the test usually rejects the null hypothesis of no bubbles for the US market. Similarly, Wu (1997) considers deviations of stock prices from the present-value model, and he assumes that dividends follow an autoregressive process. However, Wu (1997) treats the bubble as an unobservable component which he estimates with the Kalman filter. The analysis attributes large portions of stock price movements to speculative bubbles in the S&P 500. Al-Anaswah and Wilfling (2011); Lammerding et al. (2013) extend the state space model in Wu (1997) by allowing the bubble to switch between two alternative regimes, namely an explosive and a stationary regime. Al-Anaswah and Wilfling (2011) adopt the methodology of Kim and Nelson (1999) to identify regime-switching of speculative bubbles in stock price data. Differently, Lammerding et al. (2013) propose a Bayesian
approach to estimate the Markov-switching state space model of speculative bubbles in oil price data.

In line with previous research on the identification of speculative bubbles, we employ artificial as well as real-world datasets. The artificial bubble processes are defined in the sense of Evans (1991), whereas the real-world datasets are drawn from Datastream. We consider 20 years of monthly data (November 1997 - October 2017) for the dividend yield and the price index. We use data for five countries: United States, United Kingdom, Malaysia, Japan and Brazil. We have decided to conduct the analysis on this set of countries because of their economic relevance and the severe bubble episodes experienced in the past (Kindleberger and Aliber, 2003). The advantage of the artificial datasets with respect to real-world data is that the bubbles’ collapse dates are known, hence they allow us to assess the accuracy of our bubble-detection method. Instead, for the real-world datasets we rely on what economic historians have classified as bubble periods (Kindleberger and Aliber, 2003).

The main contribution of this work is the inclusion of an unobservable bubble component subject to a surviving and a collapsing regime in the present-value model by Binsbergen et al. (2010). We show that our new bubble-detection method is able to correctly identify 92.27% of all the bubble collapsing dates in the artificial datasets. Moreover, it never signals a bubble when there is none in the price process. These results represent an improvement with respect to the methodology discussed in Al-Anaswah and Willling (2011). Also, we find that our framework is able to identify most of the bubble periods classified as such by Kindleberger and Aliber (2003). Consistent with Al-Anaswah and Willling (2011) and Lammerding et al. (2013), we document the existence of statistically significant Markov-switching in the data-generating process of real-world stock price bubbles.

Our framework is also able to predict dividend growth rates as well as returns with $R^2$ values ranging from 74.07% to 78.89% for dividend growth rates and 4.04% and 20.71% for returns in the artificial datasets. In the real-world datasets, the $R^2$ values for dividend growth rates are quite high, the highest value is recorded for the US where it is equal to 70.49% while the lowest value is registered for Brazil where it is equal to 49.10%. However, the $R^2$ values for returns are less than 1% with the exception of Brazil where it is above 3%.

We show that present-value models should not ignore the bubble component of stock prices. Indeed, we find that in the surviving bubble regime most of the variation in the price-dividend ratio is related to the bubble variation. Specifically, bubble variation accounts for more than 50% of the price-dividend variation in all the countries under study with the exception of Brazil where it accounts for about 36%. Further, bubble variation
explains also a large share of unexpected return variation in the surviving bubble regime.

The paper is structured as follows: next section reviews the present-value model by Campbell and Shiller (1988). In Section 3, we present the econometric model, and Section 4 discusses the posterior sampler. In Section 5, we present the data sources and some descriptive statistics. Section 6 discusses the results, and Section 7 concludes.

## 2 Economic Model

In this section we briefly review the log-linearized present-value model in the spirit of Campbell and Shiller (1988) in which both expected returns and expected dividend growth rates are treated as latent variables as suggested by Binsbergen et al. (2010).

Let denote with $pd_t$ and $\Delta d_{t+1}$ respectively the log price-dividend ratio and the log dividend growth rate

\[ pd_t = \log \left( \frac{P_t}{D_t} \right), \]
\[ \Delta d_{t+1} = \log \left( \frac{D_{t+1}}{D_t} \right). \]

The log gross return, denoted as $r_{t+1}$, is defined as follows

\[ r_{t+1} \equiv \log \left( \frac{P_{t+1} + D_{t+1}}{P_t} \right) = \log(P_{t+1} + D_{t+1}) - \log(P_t). \]

Equation (1) is nonlinear since it involves the log of the sum of price and dividend. However, using the first order Taylor expansion it can be well approximated by

\[ r_{t+1} \simeq \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t, \]

where $\kappa$ and $\rho$ are parameters of linearizations, $\kappa = \log(1 + \exp(\bar{pd})) - \rho \bar{pd}$ and $\rho = \frac{\exp(\bar{pd})}{1 + \exp(\bar{pd})}$, $\bar{pd} = \mathbb{E}[pd]$ (Campbell and Shiller, 1988).

Iterating forward Eq. (2) and imposing the transversality condition, we obtain the unique no-bubble solution

\[ pd_t' = \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} \mathbb{E}_t[\Delta d_{t+j} - r_{t+j}], \]

5
Similar to Binsbergen et al. (2010), we assume that both expected returns ($\mu_t \equiv E_t[r_{t+1}]$) and expected dividend growth rates ($g_t \equiv E_t[\Delta d_{t+1}]$) follow an AR(1) process

$$\mu_{t+1} = \delta_0 + \delta_1(\mu_t - \delta_0) + \epsilon_{\mu_{t+1}}^\mu,$$

$$g_{t+1} = \gamma_0 + \gamma_1(g_t - \gamma_0) + \epsilon_{g_{t+1}}^g.$$  

The dividend growth rate and the return rate are respectively equal to their expected value plus an orthogonal shock:

$$\Delta d_{t+1} = g_t + \epsilon_{d_{t+1}}^d,$$

$$r_{t+1} = \mu_t + \epsilon_{r_{t+1}}^r.$$ 

Assuming that $\lim_{j \to \infty} \rho^j p d_{t+j} = 0$ and taking expectations conditional upon time $t$ we obtain the fundamental price-dividend ratio:

$$pd_t^f = \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[\Delta d_{t+j} - r_{t+j}]$$

$$= \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[g_{t+j-1} - \mu_{t+j-1}]$$

$$= \frac{\kappa}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j E_t[g_{t+j} - \mu_{t+j}]$$

$$= \frac{\kappa}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j E_t[\gamma_0 + \gamma_1^j(g_t - \gamma_0) - \delta_0 - \delta_1^j(\mu_t - \delta_0)]$$

$$= \frac{\kappa}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j E_t[\gamma_0 + \gamma_1^j(g_t - \gamma_0) - \delta_0 - \delta_1^j(\mu_t - \delta_0)]$$

$$= \frac{\kappa}{1 - \rho} + \frac{\gamma_0 - \delta_0}{1 - \rho} + \frac{g_t - \gamma_0}{1 - \rho \gamma_1} + \frac{\mu_t - \delta_0}{1 - \rho \delta_1},$$

which uses

$$E_t[x_{t+j}] = \alpha_0 + \alpha_1^j(x_t - \alpha_0),$$

$$\text{where} \quad \alpha_1^j = \frac{\gamma_1^j}{1 - \rho \gamma_1}.$$
provided that

\[ x_{t+1} = \alpha_0 + \alpha_1^j(x_t - \alpha_0) + \epsilon_{t+1}. \]  

(10)

Finally, the fundamental price-dividend ratio can be written

\[ pd_t^f = A - B_1 \hat{\mu}_t + B_2 \hat{g}_t, \]

where \[ A = \frac{\kappa - \delta_0 + \gamma_0}{1 - \rho}, \]
\[ B_1 = \frac{1}{1 - \rho \delta_1}, \]
\[ B_2 = \frac{1}{1 - \rho \gamma_1}, \]
\[ \hat{\mu}_t = \mu_t - \delta_0, \]
\[ \hat{g}_t = g_t - \gamma_0. \]

(11)

It is important to stress that if the transversality solution does not hold, the no-bubble solution \( pd_t^f \) in (11) represents only a particular solution to the difference equation (2), and the general solution has the form

\[ pd_t = pd_t^f + b_t. \]

(12)

where \( b_t \) is a rational speculative bubble, that is a deviation of the stock price from fundamentals generated by extraneous factors or rumors and driven by self-fulfilling expectations. The bubble component of the price-dividend ratio satisfies the homogeneous difference equation

\[ \mathbb{E}_t[b_{t+1}] = \frac{b_t}{\rho}. \]

(13)

In line with the literature (i.e., Wu, 1997; Al-Anaswah and Willling, 2011; Lammerding et al., 2013), we assume that the bubble component follows a linear AR(1) process

\[ b_{t+1} = \frac{b_t}{\rho} + \epsilon_{t+1}^b, \quad \epsilon_{t+1}^b \sim N(0, \sigma_b^2). \]

(14)

When estimating the price-dividend equation (12), we are confronted with the problem that expected returns, expected dividend growth rates, and the bubble component are unobservables. Hence, we have to express our model in state space form.
3 Econometric Model

Bubbles are empirically plausible only if they are likely to collapse after reaching high levels. For instance, Al-Anaswah and Wilfling (2011) and Lammerding et al. (2013) allow the bubble in the present-value model in Wu (1997) to switch between two alternative regimes: an explosive and a stationary regime. Using both stock and oil price data, they document statistically significant Markov-switching between these two regimes. Their findings motivate us to extend the present-value model of Binsbergen et al. (2010) to incorporate a speculative bubble that switches between two regimes. The two regimes aim to represent the two distinct phases in the bubble process, namely, one in which the bubble survives and one in which it collapses.

Thus, we allow the model parameters to switch between two distinct regimes $S_t \in \{1, 2\}$. The regime indicator $S_t$ is governed by a first-order Markov process with constant transition probabilities,

$$
\Pi = \begin{pmatrix}
    p_{11} & 1 - p_{11} \\
    1 - p_{22} & p_{22}
\end{pmatrix},
$$

where $p_{11}$ and $p_{22}$ are between 0 and 1.

The model transition equations can be written as:

$$
\begin{align*}
\hat{g}_t &= \gamma_{1,S_{t+1}} \hat{g}_{t-1} + \epsilon^g_t, \\
\hat{\mu}_t &= \delta_{1,S_{t+1}} \hat{\mu}_{t-1} + \epsilon^\mu_t, \\
b_t &= 1/\rho_{S_{t+1}} b_{t-1} + \epsilon^b_t.
\end{align*}
$$

(15)

The dividend growth rate is then equal to

$$
\Delta d_{t+1} = \gamma_{0,S_{t+1}} + \hat{g}_t + \epsilon^d_{t+1},
$$

(16)

and the price-dividend equation is

$$
p d_{t+1} = A_{S_{t+1}} + B_{2,S_{t+1}} \hat{g}_{t+1} - B_{1,S_{t+1}} \hat{\mu}_{t+1} + b_{t+1} \\
= A_{S_{t+1}} + B_{2,S_{t+1}} \tilde{\gamma}_{1,S_{t+1}} \hat{g}_t - B_{1,S_{t+1}} \tilde{\delta}_{1,S_{t+1}} \hat{\mu}_t + 1/\tilde{\rho}_{S_{t+1}} b_t + B_{2,S_{t+1}} \tilde{\epsilon}^g_{t+1} \\
- B_{1,S_{t+1}} \tilde{\epsilon}^\mu_{t+1} + \tilde{\epsilon}^b_{t+1} + \tilde{\epsilon}^e_{t+1},
$$

(17)

where, for $i, j \in \{1, 2\}$ and $j \neq i$, we have defined:

$$
\begin{align*}
\tilde{\gamma}_{1,S_{t+1}} &= \mathbb{E}_{t+1}[\gamma_{1,S_{t+2}} | S_{t+1} = i] = p_{ii} \gamma_{1,i} + (1 - p_{ii}) \gamma_{1,j}, \\
\tilde{\delta}_{1,S_{t+1}} &= \mathbb{E}_{t+1}[\delta_{1,S_{t+2}} | S_{t+1} = i] = p_{ii} \delta_{1,i} + (1 - p_{ii}) \delta_{1,j}, \\
\tilde{\rho}_{S_{t+1}} &= \mathbb{E}_{t+1}[\rho_{S_{t+2}} | S_{t+1} = i] = p_{ii} \rho_{1,i} + (1 - p_{ii}) \rho_{1,j}.
\end{align*}
$$

(18)
Notice that we have added to the equation for the price-dividend an orthogonal error $e_{t+1}^e$. Indeed when we substitute the transition variables at $t + 1$, we are confronted with the fact that next period regime is unknown. Hence, we use their expectation conditioned on the information available at time $t + 1$ which generate an error, measured by $e_{t+1}^e$.

In order to circumvent nonstationarity problems, we express the price-dividend equation in first difference:

$$
\Delta p_{d,t+1} = A_{S_{t+1}} - A_{S_t} + (B_{2,S_{t+1}} \delta_{1,S_{t+1}} - B_{2,S_t}) \hat{g}_t - (B_{1,S_{t+1}} \delta_{1,S_{t+1}} - B_{1,S_t}) \hat{\mu}_t + (1/\hat{\rho}_{S_{t+1}} - 1)b_t + B_{2,S_{t+1}} e_{t+1}^g - B_{1,S_{t+1}} e_{t+1}^\mu + e_{t+1}^b + e_{t+1}^e. 
$$

Concerning the return process, approximation (2) together with equation (12) imply that the return shock has the following form:

$$
\epsilon_{t+1}^r = \epsilon_{t+1}^d + \rho \epsilon_{t+1}^{pd},
$$

where $\epsilon_{t+1}^{pd} = B_2 \epsilon_{t+1}^g - B_1 \epsilon_{t+1}^\mu + \epsilon_{t+1}^b$. Since the series of market returns is fully described by the dividend growth rates and the price-dividend ratio, we omit it from our state space specification. Let $\alpha_t$ denote the $7 \times 1$ vector of unobservable variables, and $y_t$ be the $2 \times 1$ vector of observable variables:

$$
\alpha_{t+1} = \begin{pmatrix} \hat{g}_t & \hat{\mu}_t & b_t & e_{t+1}^g & e_{t+1}^\mu & e_{t+1}^d & e_{t+1}^b \end{pmatrix}, \quad y_{t+1} = \begin{pmatrix} \Delta d_{t+1} & \Delta p_{d,t+1} \end{pmatrix}'.
$$

We can now express the model in matrix form:

$$
\alpha_{t+1} = G_{S_{t+1},S_t} \alpha_t + \Gamma \xi_{t+1},
$$

$$
y_{t+1} = M_{S_{t+1},S_t} + Z_{S_{t+1},S_t} \alpha_{t+1} + \eta_{t+1},
$$

where $\alpha_t \sim N(0, \Gamma V \Gamma')$, $G$, $M$, and $Z$ are time invariant matrices of the appropriate dimensions, and $\xi_t$ and $\eta_t$ are $(4 \times 1)$ and $(2 \times 1)$ vector of disturbances, respectively

$$
\xi_{t+1} = \begin{pmatrix} \epsilon_{t+1}^g & \epsilon_{t+1}^\mu & \epsilon_{t+1}^d & \epsilon_{t+1}^b \end{pmatrix}', \quad \eta_{t+1} = \begin{pmatrix} 0 & \epsilon_{t+1}^e \end{pmatrix}',
$$

with

$$
\xi_t \sim N(0, V_{S_t}),
$$

$$
\eta_t \sim N(0, R_{S_t}).
$$
The model matrices are defined as follows:

\[
G_S = \begin{pmatrix}
\gamma_{1,S_t} & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & \delta_{1,S_t} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1/\rho_{S_t} & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

\[
\Gamma = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix},
\]

\[
\Omega_{S_t} = \begin{pmatrix}
\sigma_{g,S_t}^2 & \sigma_{g\mu,S_t} & \sigma_{gd,S_t} \\
\sigma_{g\mu,S_t} & \sigma_{\mu,S_t}^2 & \sigma_{\mu d,S_t} \\
\sigma_{gd,S_t} & \sigma_{\mu d,S_t} & \sigma_{d,S_t}^2 \\
\end{pmatrix},
\]

\[
V_{S_t} = \begin{pmatrix}
\Omega_{S_t} & 0 \\
0 & \sigma_{b,S_t}^2 \\
\end{pmatrix},
\]

\[
R_{S_t} = \begin{pmatrix}
0 & 0 & 0 \\
0 & \sigma_{e,S_t}^2 \\
0 & 0 & \sigma_{e,S_t}^2 \\
\end{pmatrix},
\]

\[
M_{S_t,S_{t-1}} = \begin{pmatrix}
\gamma_{0,S_t} & 0 & 0 & 0 & 1 & 0 \\
A_{S_t} - A_{S_{t-1}} & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

\[
Z_{S_t,S_{t-1}} = \begin{pmatrix}
1 & 0 & 0 & 1 \\
B_{2,S_t} & \tilde{\gamma}_{1,S_t} - B_{2,S_{t-1}} & -B_{1,S_t} & 0 & 1 \\
\end{pmatrix}.
\]

Given that the bubble process is exogenous, we have assumed \(\sigma_{gb} = \sigma_{\mu b} = \sigma_{gb} = 0\).

### 4 Bayesian Estimation

In this section we describe a Bayesian approach for estimating our Markov-switching state space model. Since the number of model parameters is quite large, standard maximum likelihood estimation tends to be numerically unstable and may result in estimates that are locally, but not globally, maximal. For this reason we apply MCMC methods which are numerically more robust. A key novel feature of our approach is that it builds upon the band and sparse matrix algorithms for state space models developed in Chan and Jeliazkov (2009), McCausland et al. (2011) and Chan (2013), which are shown to be more efficient than conventional Kalman filter-based algorithms.

In what follows we use the index \(i\) to denote the regime, \(i \in \{1, 2\}\). There are two sets of regime-specific parameters. When it does not cause confusion, we would drop the regime index \(i\). For estimation, we split the latent states and model parameters into 7 blocks: states \(\alpha\), covariance matrices \(\Omega_i\), variances \((\sigma_{b,i}^2, \sigma_{e,i}^2)\), parameters \(\Theta_1 = (\gamma_{0,1}, \delta_{0,1}, \gamma_{0,2}, \delta_{0,2})\), \(\Theta_2 = (\rho_1, \gamma_{1,1}, \delta_{1,1}, \rho_2, \gamma_{1,2}, \delta_{1,2})\)\(^1\), regime indicators \(S\), and Markov regime-switching probabilities \(p_{11}\) and \(p_{22}\).

We assume the following prior distributions: i. \(\Omega_i \sim IW(\nu_{01}, S_{01})\); ii. \(\sigma_{k,i}^2 \sim IG(\nu_{02}, S_{0k})\),

---

\(^1\)The parameter of linearization \(\kappa\) is expressed as a function of \(\rho\): \(\kappa(\rho) = \log(1 + \exp(\bar{pd})) - \rho \bar{pd}\), where \(\bar{pd}\) is the unconditional expected price-dividend ratio. We set it equal to the sample average of the price-dividend ratio of each dataset.
\( k = \{b, e\}; \ iii. \ \Theta_1 \sim N(\Theta_1, V_{\Theta_1}); \ iv. \ \Theta_2 \sim N(\Theta_2, V_{\Theta_2}); \ v. \ p_{11} \sim Beta(u_{11}, u_{12}), \) and \( p_{22} \sim Beta(u_{22}, u_{21}) \). For brevity, we use \( \Theta \) to denote the vector \((\Theta_1, \Theta_2)\).

We define Regime 1 as the bubble surviving regime, while Regime 2 represents the bubble collapsing regime. The main model parameter is \( \rho \), which governs the growth rate of the bubble process. When \( \rho \) increases, the bubble’s growth rate decreases. In particular, when \( \rho \leq 1 \), the bubble is explosive; when \( \rho > 1 \), the bubble follows a stationary AR(1) process. We assume a Normal prior for \( \rho \) with mean equal to 0.75 in Regime 1 and 1.25 in Regime 2. For other parameters we assume the same priors across the two regimes. The values of the hyperparameters are informed by the the estimation results of previous studies (Binsbergen et al., 2010; Choi et al., 2017; Piatti and Trojani, 2017). Finally, we adopt a conjugate prior \( Beta(15, 1) \) for the transition probabilities \( p_{11} \) and \( p_{22} \). Table 1 summarizes the priors and starting values for the MCMC algorithm.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega )</td>
<td>( IW(3 + 2, 0.01 \times I_3) )</td>
<td>( IW(3 + 2, 0.01 \times I_3) )</td>
</tr>
<tr>
<td>( \sigma^2_b )</td>
<td>( IG(5, 0.04) )</td>
<td>( IG(5, 0.04) )</td>
</tr>
<tr>
<td>( \sigma^2_e )</td>
<td>( IG(5, 0.0004) )</td>
<td>( IG(5, 0.0004) )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( N(0.75, 0.05^2) )</td>
<td>( N(1.25, 0.05^2) )</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>( N(0.00, 0.05^2) )</td>
<td>( N(0.00, 0.05^2) )</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>( N(0.50, 0.05^2) )</td>
<td>( N(0.50, 0.05^2) )</td>
</tr>
<tr>
<td>( \delta_0 )</td>
<td>( N(0.02, 0.05^2) )</td>
<td>( N(0.02, 0.05^2) )</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>( N(0.80, 0.05^2) )</td>
<td>( N(0.80, 0.05^2) )</td>
</tr>
<tr>
<td>( p_{11}, p_{22} )</td>
<td>( Beta(15, 1) )</td>
<td>( 0.800 )</td>
</tr>
</tbody>
</table>

We implement the following 7-block Metropolis-within-Gibbs sampler to simulate from the joint posterior distribution:

1. Sample from \( f(\alpha|Y, \Theta, \Omega, \sigma^2_b, \sigma^2_e, S, p_{11}, p_{22}) \).

It can be shown that the full conditional distribution of \( \alpha \) is Gaussian. As a first step, we rewrite the transition and the measurement equations in matrix form:

\[
H_G \alpha = \hat{\Gamma} \xi, \ \hat{\Gamma} \xi \sim N(0, W),
\]

\[
Y = \tilde{M} + H_Z \alpha + \eta, \ \eta \sim N(0, \Phi),
\]

where
\[ H_G = \begin{pmatrix} I_7 & & \\ -G_{S_2} & I_7 & \\ & \ddots & \ddots \\ & & -G_{S_T} & I_7 \end{pmatrix}, \quad W^2 = \begin{pmatrix} \Gamma V_{S_1} \Gamma' \\ \Gamma V_{S_2} \Gamma' \\ \vdots \\ \Gamma V_{S_T} \Gamma' \end{pmatrix} \]

\[ H_Z = \begin{pmatrix} Z_{S_1, S_0} & \cdots \\ Z_{S_2, S_1} & \ddots \\ \vdots & \ddots & \ddots \\ Z_{S_T, S_{T-1}} & \cdots \\ \end{pmatrix}, \quad \Phi = \begin{pmatrix} R_{S_1} \\ R_{S_2} \\ \vdots \\ R_{S_T} \end{pmatrix} \]

\[ \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_T \end{pmatrix}, \quad \tilde{\Gamma} = \begin{pmatrix} \Gamma \\ \vdots \\ \Gamma \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi_1 \\ \vdots \\ \xi_T \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix}, \quad \tilde{M} = \begin{pmatrix} M_{S_1, S_0} \\ \vdots \\ M_{S_T, S_{T-1}} \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_T \end{pmatrix}. \]

Then, the conditional posterior \([\alpha|Y, \Theta, \Omega, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22}] \sim N(\hat{\alpha}, P^{-1})\), where

\[ P = H_G' W^{-1} H_G + H_Z' \Phi^{-1} H_Z, \]

\[ \hat{\alpha} = P^{-1}(H_Z' \Phi^{-1}(Y - \tilde{M})). \tag{24} \]

To simulate from \(N(\hat{\alpha}, P^{-1})\), we first obtain the Cholesky factor \(C\) of \(P\) such that \(C'C = P\). Then, given \(u \sim N(0, I)\), we solve \(Cx = u\) for \(x\) by back substitution and take \(\alpha = \hat{\alpha} + x\). It can be shown that \(\alpha \sim N(\hat{\alpha}, P^{-1})\); see, e.g., Chan and Jeliazkov (2009).

2. Sample from \(f(\Omega|y, \alpha, \Theta, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22})\). This step is standard as \(\Omega\) follows an Inverse-Wishart distribution:

\[ [\Omega_t|y, \alpha, \Theta, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22}] \sim IW\left(\nu_{01} + \sum_{t=1}^{T} \mathbb{I}(S_t = i), S_{01} + \sum_{t=1}^{T} (\epsilon_t^g \epsilon_t^d) \mathbb{I}(S_t = i)\right), \]

where \(IW\) stands for the Inverse-Wishart distribution, \(\epsilon_t = (\epsilon_t^g, \epsilon_t^d)\).

3. Sample from \(f(\sigma^2_{k,i}|y, \alpha, \Theta, \Omega, S, p_{11}, p_{22})\), \(k = \{b, e\}\). This step is standard as each of the variances follows an Inverse-Gamma distribution:

\[ [\sigma^2_{k,i}|y, \alpha, \Theta, \Omega, S, p_{11}, p_{22}] \sim IG\left(\nu_{02} + \sum_{t=1}^{T} \mathbb{I}(S_t = i), S_{0k} + \sum_{t=1}^{T} (\epsilon_t^g \epsilon_t^d) \mathbb{I}(S_t = i)\right), \]

where \(IG\) stands for the Inverse-Gamma distribution.

\[ ^2\text{Note that the first three diagonal elements of } \Gamma V_{S_i} \text{ are zero, hence matrix } W \text{ is singular. We substitute the zero elements with } 10^{-8} \text{ in order to preserve the invertibility of } W. \]
4. Sample from $f(\Theta_1|y, \alpha, \Theta_2, \Omega, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22})$. This step is also standard as $\Theta_1$ follows a Normal distribution. To see that, we first write the measurement equation as

$$y_t = M_{S_t, S_{t-1}} + (Z_{S_t, S_{t-1}} G_{S_t})\alpha_{t-1} + Z_{S_t, S_{t-1}} \Gamma\xi_t + \eta_t.$$  

(25)

We can then express the constant term $M_{S_t, S_{t-1}}$ as $M_{S_t, S_{t-1}} = C_{S_t, S_{t-1}} + X_{S_t, S_{t-1}} \Theta_1$, where $C_{S_t, S_{t-1}} = (0, \kappa(\rho_{S_t})/(1 - \rho_{S_t}) - \kappa(\rho_{S_{t-1}})/(1 - \rho_{S_{t-1}}))'$,

$$X_{1,1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad X_{1,2} = \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}, \quad X_{2,1} = \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix}.$$  

Using standard linear regression results, one can show that the conditional posterior is $[\Theta_1|y, \alpha, \Theta_2, \Omega, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22}] \sim N(\hat{\Theta}_1, K_{\Theta_1})$, where

$$K_{\Theta_1} = (V_{\Theta_1}^{-3} + \bar{X}'\Sigma^{-1}\bar{X})^{-1}, \quad \hat{\Theta}_1 = K_{\Theta_1}(V_{\Theta_1}^{-1}\Theta_1 + \bar{X}'\Sigma^{-1}(Y - \tilde{C} - H_{ZG}\alpha)).$$  

(26)

The matrix $\bar{X}$ is a $2T \times 4$ matrix $\bar{X} = (X_{S_1, S_0}, \ldots, X_{S_T, S_{T-1}})'$, $\tilde{C} = (C_{S_1, S_0}, \ldots, C_{S_T, S_{T-1}})'$, and $Y, \alpha$ are the stacked vectors of $y_t$ and $\alpha_t$ respectively, $H_{ZG}$ and $\Sigma$ are defined as:

$$H_{ZG} = \begin{pmatrix} 0 \\ Z_{S_2, S_1} G_{S_2} \\ \vdots \\ Z_{S_T, S_{T-1}} G_{S_T} \end{pmatrix}, \quad \Sigma^3 = \begin{pmatrix} (Z_{S_1, S_0}(\Gamma V_{S_1} \Gamma')Z_{S_1, S_0}^T + R_{S_1}) \\ \vdots \\ (Z_{S_T, S_{T-1}}(\Gamma V_{S_T} \Gamma')Z_{S_T, S_{T-1}}^T + R_{S_T}) \end{pmatrix}.$$  

5. Sample from $f(\Theta_2|Y, \alpha, \Theta_1, \Omega, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22})$. Since this conditional distribution is nonstandard, we sample $\Theta_2$ using an adaptive Random Walk Metropolis-Hastings algorithm (Roberts and Rosenthal, 2009). In particular, we update each element of $\Theta_2$ at a time. Given the current draw $\Theta_2^{(s)}$, we update the $j$-th variable by adding a normal random variable centered at zero to obtain the candidate draw $\Theta_2^{(s+1)}$.\footnote{The matrix $\Gamma V_0 \Gamma'$ is singular as the first three diagonal elements are zero. To avoid numerical problems, we substitute them with $10^{-8}$.} The

\text{For the first batch of 50 iterations, we update each variable $j$ by adding a $N(0, 0.1^2)$ distributed random variable. Then, after the $l$-th batch of 50 iterations, we update the logarithm of the standard...}
candidate is then accepted with probability
\[ a(\Theta_2^{(s)}; \Theta_2^*) = \min \left\{ \frac{f(\Theta_2 = \Theta_2^*|Y, \alpha, \Theta_1, \Omega, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22})}{f(\Theta_2 = \Theta_2^{(s)}|Y, \alpha, \Theta_1, \Omega, \sigma_b^2, \sigma_e^2, S, p_{11}, p_{22})}, 1 \right\}. \tag{27} \]

We impose stationarity conditions for expected dividend growth rates and expected returns, i.e., \(-1 < \gamma_1 < 1, -1 < \delta_1 < 1\).

6. Sample from \( f(S|Y, \alpha, \Theta, \Omega, \sigma_b^2, \sigma_e^2, p_{11}, p_{22}) \). This step can be done using the algorithm proposed by Chib (1996); see also Kim and Nelson (1999). Specifically, we use the following decomposition of the joint conditional density:\footnote{For notational convenience, in what follows we suppress the dependence on the model parameters.}
\[ f(S|Y, \alpha) = f(S_T|Y, \alpha) \prod_{t=1}^{T-1} f(S_t|S_{t+1}, Y_{1:t}, \alpha_{1:t}), \tag{28} \]
where \( Y_{1:t} \) denotes all the data up to time \( t \), and \( \alpha_{1:t} \) is similarly defined.

To compute each of these conditional distributions, we first run the Hamilton filter (Hamilton, 1989) to get the filtered distributions \( f(S_t|Y_{1:t}, \alpha_{1:t}), t = 1, 2, ..., T \). The last iteration of the filter provides \( f(S_T|Y, \alpha) \). More specifically, these filtered distributions are defined by
\[ f(S_t|Y_{1:t}, \alpha_{1:t}) \propto f(y_t|S_t, \alpha_{t-1}, y_{t-1})f(S_t|Y_{1:t-1}, \alpha_{1:t-1}), \]
where \( f(y_t|S_t, \alpha_{t-1}, y_{t-1}) \) is a multivariate normal distribution defined by the model. Then, the conditional distribution \( f(S_t|S_{t+1}, Y_{1:t}, \alpha_{1:t}) \) can be computed by using:
\[ f(S_t|S_{t+1}, Y_{1:t}, \alpha_{1:t}) \propto f(S_{t+1}|S_t)f(S_t|Y_{1:t}, \alpha_{1:t}), \]
where \( f(S_{t+1}|S_t) \) is the transition probability and \( f(S_t|Y_{1:t}, \alpha_{1:t}) \) is calculated using the Hamilton filter as described above. Note that the probability \( Pr(S_t = 2|S_{t+1}, Y_{1:t}, \alpha_{1:t}) \) can be obtained after the normalization:
\[ Pr(S_t = 2|S_{t+1}, Y_{1:t}, \alpha_{1:t}) = \frac{f(S_{t+1}|S_t = 2)f(S_t = 2|Y_{1:t}, \alpha_{1:t})}{\sum_{j=1}^{2} f(S_{t+1}|S_t = j)f(S_t = j|Y_{1:t}, \alpha_{1:t})}. \tag{29} \]

Finally, to obtain a draw from \( f(S_t|S_{t+1}, Y_{1:t}, \alpha_{1:t}) \), we generate a random number from a uniform distribution between 0 and 1. If the generated number is less than 0.44, we increase \( \log(s_j) \) by \( \delta(l) \); otherwise we decrease it by the same amount. Note that Roberts et al. (1997) and Roberts et al. (2001) show that in various one-dimensional settings the optimal acceptance rate is around 0.44.
$Pr(S_t = 2|S_{t+1}, Y_{t+1}, \alpha_{t+1})$, we set $S_t = 2$; otherwise we set it equal to 1.

7. Sample from $f(p_{11}, p_{22}|Y, \alpha, \Theta, \Omega, \sigma_\delta^2, \sigma_\epsilon^2, S)$.

Conditional on $S$, the transition probabilities $p_{11}$ and $p_{22}$ are independent of the data $y$, the state variables $\alpha$, and other model parameters.

Since we choose conjugate priors for both $p_{11}$ and $p_{22}$, we only need to calculate the number of switches between the regimes in order to derive the posterior distributions of $p_{11}$ and $p_{22}$. The posterior distributions are two independent beta distributions:

$$p_{11} \sim \text{Beta}(u_{11} + n_{11}, u_{12} + n_{12}), \quad p_{22} \sim \text{Beta}(u_{22} + n_{22}, u_{21} + n_{21}),$$

where $n_{ij}$ refers to the transitions from state $i$ to $j$; $i, j \in \{1, 2\}$.

5 Data and Descriptive Statistics

We apply our two-regime Markov-switching state space model to artificial as well as real-world datasets. The artificial bubble processes are defined in the sense of Evans (1991), whereas the real-world datasets are drawn from Datastream. We consider 20 years of monthly data (November 1997 - October 2017) for the dividend yield and the price index. We use data for five countries: United States (US), United Kingdom (UK), Malaysia (MY), Japan (JP) and Brazil (BR). We choose this set of countries because of their economic relevance and the severe bubble episodes experienced in the past (Kindleberger and Aliber, 2003). The baseline model of Binsbergen et al. (2010) is built on the approximation in (2). Hence, to conduct our analysis we need the approximation error to be sufficiently small and not persistent. For this reason we focus only on the last twenty years of data, for this period the average approximation error ranges from $-0.0065$ for Brazil to $-0.0001$ for the US and the UK. If we use the entire sample available, the approximation errors increase substantially. For example, the average approximation errors rise to $-0.0115$ and $-0.0082$ for the US and the UK respectively. The approximation errors are particularly high in the first part of the sample, implying that it is not appropriate to apply the model to the entire sample of data.
5.1 Simulated Datasets

Evans (1991) considers a class of rational bubbles that are positive and periodically collapsing defined as follows:

\[
B_{t+1} = \begin{cases} 
\frac{B_t}{1 + r} u_{t+1} & \text{if } B_t \leq \lambda \\
\left( \delta + \frac{1 + r}{\pi} (B_t - \frac{\delta}{1 + r}) \theta_{t+1} \right) u_{t+1} & \text{if } B_t > \lambda,
\end{cases}
\]  

(30)

where \( \delta \) and \( \lambda \) are real positive parameters such that \( 0 < \delta < \lambda(1 + r) \). The variable \( u_t \) is assumed to be independent and identically distributed (iid) lognormally with unit mean. Specifically, we assume \( u_t = \exp(x_t - \tau^2/2) \) with \( x_t \) iid \( N(0, \tau^2) \). The process \( \theta_t \) is an exogenous iid Bernoulli process which assumes the value 1 with probability \( \pi \) and the value 0 with probability \( 1 - \pi \), \( 0 < \pi \leq 1 \). The parameters \( \delta \), \( \lambda \), and \( \pi \) govern the frequency with which bubbles erupt, the scale of the bubble and the average length of time before collapse.

As long as the bubble process is below \( \lambda \), the bubble grows at the mean rate \( 1 + r \). When \( B_t > \lambda \) the bubble grows at the faster mean rate \( (1 + r)\pi^{-1} \) and it can collapse with a probability \( 1 - \pi \) in each period. Whenever the bubble collapses, it falls to \( \delta \) and the process starts again. Panel A of Table 2 specifies the parameters of the Evans-bubble process.

We simulate expected returns and expected dividend growth rates according to equations (4) and (5) respectively. We use equations (6) and (11) to generate the dividend growth rate and the fundamental price-dividend ratio series. We generate the bubble stock price-dividend ratio by adding the logarithm of the Evans-bubble (30) to the fundamental price-dividend ratio:

\[
pd_t = pd^f_t + \log(B_t).
\]  

(31)

Figure 1 shows a realization for the log-price-dividend ratio and the log-Evans-bubble.

Finally, the time series of returns is built from the series of the dividend growth rate and the price-dividend ratio using approximation (2). In Panel B of Table 2 we present the parameters used for generating the time-series of our state space model. We generate five artificial datasets with either 100 or 200 observations, for space constraint we report only the results for the biggest datasets of 200 observations.


Table 2: Parameter specification for artificial datasets

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Evans-bubble process parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\tau^2$</td>
<td>0.0025</td>
</tr>
<tr>
<td>$r$</td>
<td>0.0500</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5000</td>
</tr>
<tr>
<td>$B_0$</td>
<td>0.5000</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.5000</td>
</tr>
<tr>
<td># observations</td>
<td>100 or 200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Panel B: state space model parameters</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>0.0500</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.0100</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.8600</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5000</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.6000</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.7000</td>
</tr>
<tr>
<td>$\sigma_1^2$</td>
<td>0.0010</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>0.0020</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
<td>0.0013</td>
</tr>
<tr>
<td>$\sigma_{g\mu}$</td>
<td>0.0009</td>
</tr>
<tr>
<td>$\sigma_{g\phi}$</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

Panel A reports the parameter specification for the Evans-bubble in (30). Panel B reports the parameters used for generating the time-series from equations (4), (5), (6), and (11).

![Figure 1: Artificial dataset 1](image1)

5.2 Real-World Datasets

We use monthly data on the dividend yield and the price index from five countries: United States (US), United Kingdom (UK), Malaysia (MY), Japan (JP), and Brazil (BR). The sample period is from November 1997 to October 2017, a total of 20 years of monthly data. All the data are sourced from Datastream.

Table 3 reports some descriptive statistics for the log-price-dividend ratio of the country indices. The US, the UK and Japan have an average log-price-dividend ratio of about...
6.50, while the average for Malaysia is 5.08 and 4.26 for Brazil, with the latter being more volatile. The correlations among the developed countries (US, UK and Japan) are quite high, and they range from 0.75 between the US and the UK, to 0.52 between Japan and the UK. The price-dividend ratio of Malaysia is positively correlated with the other countries with the exception for Japan where the correlation is negative. The price-dividend ratio of Brazil is negatively correlated to Japan and it shows almost no correlation with the US.

Figure 2 shows the time series plot of the price-dividend ratio for the five country indices. The series for Japan hits a maximum of 3000 in year 2000, then it decreases and it maintains levels comparable to those of the US and the UK. The price-dividend ratio for Malaysia and Brazil is lower compared to those of the other economies, and they are highly correlated. We can observe that all the countries have experienced a sharp drop in the price-dividend ratio in 2008 in correspondence with the Global Financial Crises.

<table>
<thead>
<tr>
<th>United States</th>
<th>United Kingdom</th>
<th>Malaysia</th>
<th>Japan</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>6.5562</td>
<td>6.5585</td>
<td>5.0764</td>
<td>6.6452</td>
</tr>
<tr>
<td>Std</td>
<td>0.3225</td>
<td>0.2980</td>
<td>0.4924</td>
<td>0.4847</td>
</tr>
<tr>
<td>Range</td>
<td>1.9831</td>
<td>1.6619</td>
<td>2.8625</td>
<td>2.4947</td>
</tr>
<tr>
<td>Num.obs.</td>
<td>240</td>
<td>240</td>
<td>240</td>
<td>240</td>
</tr>
</tbody>
</table>

Panel B: Correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>MY</th>
<th>JP</th>
<th>BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.7563</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MY</td>
<td>0.3726</td>
<td>0.4361</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JP</td>
<td>0.5911</td>
<td>0.5195</td>
<td>-0.1743</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>BR</td>
<td>-0.0257</td>
<td>0.3348</td>
<td>0.5490</td>
<td>-0.3843</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

This table presents summary statistics and correlations for the price-dividend ratio (in log) of the United States, United Kingdom, Malaysia, Japan and Brazil.

Figure 2: Price-dividend ratio by country
6 Results

We estimate the present-value model in equations (15) to (17) using the MCMC method described in Section 4. We run a total of 1,000,000 iterations, and discard the first 100,000 as burn-in. Using the sample of the posterior draws, we report the sample means as point estimates. 95% credible intervals are constructed using the 2.50% and 97.50% sample quantiles.

The parameter $\rho$ is the main variable of interest, indeed $1/\rho$ is the autoregressive parameter of the bubble process. A value of $\rho$ less than one implies that the bubble is explosive, while a value greater than one means that the bubble process is stable. $p_{11} = P(S_t = 1|S_{t-1} = 1)$ and $p_{22} = P(S_t = 2|S_{t-1} = 2)$ are the transition probabilities of the two-regime Markov process. If $p_{11} > p_{22}$, then the bubble surviving regime is more persistent than the bubble collapsing regime.

Table 4 reports the posterior estimates of the parameters for our state space model for the artificial datasets. The main variables are presented in bold characters. Table 5 shows the bubble 95% credible intervals for the parameters $\rho_1$ and $\rho_2$, we can observe that the two parameters are statistically significant and significantly different from each other. In particular, $\rho_1$ is significantly less than one, while $\rho_2$ is significantly greater than one.
meaning that we correctly find significant regime-switch in our artificial datasets. Moreover, in Regime 1 the bubble is explosive while in Regime 2 it collapses. The transition probability $p_{11}$ is always greater than $p_{22}$, suggesting that the surviving bubble regime is more persistent than the collapsing bubble regime.

$$p_{11} = P(S_t = 1 | S_{t-1} = 1) \quad \text{and} \quad p_{22} = P(S_t = 2 | S_{t-1} = 2)$$

Table 6: Parameter estimates - Real-world datasets

<table>
<thead>
<tr>
<th>Regimes</th>
<th>United States</th>
<th>United Kingdom</th>
<th>Malaysia</th>
<th>Japan</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>0.0025</td>
<td>0.0007</td>
<td>0.0037</td>
<td>0.0011</td>
<td>0.0013</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.0174</td>
<td>0.0193</td>
<td>0.0163</td>
<td>0.0189</td>
<td>0.0188</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9329</td>
<td>1.2459</td>
<td>0.9236</td>
<td>1.2380</td>
<td>0.9987</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.5665</td>
<td>0.4913</td>
<td>0.5568</td>
<td>0.4914</td>
<td>0.5535</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.7718</td>
<td>0.7872</td>
<td>0.7521</td>
<td>0.7762</td>
<td>0.8651</td>
</tr>
<tr>
<td>$p_{11}$, $p_{22}$</td>
<td>0.9915</td>
<td>0.9159</td>
<td>0.9905</td>
<td>0.9061</td>
<td>0.9840</td>
</tr>
</tbody>
</table>

We present the estimation results of the present-value model in equations (15) to (17) for the five real-world datasets. The model is estimated according to the procedure described in Section 4. Note that $1/\rho$ is the autoregressive parameter of the bubble process. When $\rho$ is less than one the bubble is explosive, when it is greater than one the bubble is stable.

Table 7: 95% credible intervals - Real-world datasets

<table>
<thead>
<tr>
<th>Regimes</th>
<th>United States</th>
<th>United Kingdom</th>
<th>Malaysia</th>
<th>Japan</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>0.8253</td>
<td>0.9982</td>
<td>0.8188</td>
<td>0.9924</td>
<td>0.7908</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>1.1119</td>
<td>1.3783</td>
<td>1.0997</td>
<td>1.3732</td>
<td>1.0919</td>
</tr>
</tbody>
</table>

We present the 95% credible intervals for the bubble parameter $\rho$ in Regime 1 ($\rho_1$) and Regime 2 ($\rho_2$). The 95% credible intervals consists of the 2.50% and 97.50% quantiles of the posterior distribution of $\rho_1$ and $\rho_2$. We present the 95% credible intervals for the autoregressive bubble parameter $\rho$. Again, we observe $\rho$ to be significantly different in the two regimes for all the real-world datasets except Malaysia. Moreover, we find that $\rho_1$ is significantly lower than one and $\rho_2$ is significantly greater than one at the 5%-level for the US, the UK and Brazil. Instead, in Malaysia and Japan $\rho_1$ does not appear to be significantly smaller than 1 at 5%-level, meaning that we do not find statistical significance of an explosive behaviour of the bubble process in these countries. Consistent with Al-Anaswah and Wilfling (2011), and Lammerding et al. (2013), we find that the transition probability $p_{11}$ is always greater than $p_{22}$, meaning that the surviving bubble regime is more persistent than the collapsing bubble regime in all the real-world datasets. In particular, the probability of remaining in Regime 1 ($p_{11}$) is higher than 99% in the US, the UK and Japan, while in Malaysia it is 98.40% and it equals 97.23% in Brazil. The estimates of the transition probability $p_{22}$, instead, vary between 80.29% (Brazil) and 92.89% (Japan). These results also imply that the expected duration of the surviving regime $1/(1-p_{11})$ is higher than the expected duration of the
collapsing regime $1/(1 - p_{22})$. Concerning the remaining model parameters, we find that they are not significantly different across the two regimes. The unconditional expected log dividend growth rate ($\gamma_0$) and the unconditional expected log return ($\delta_0$) are not significantly different from zero. Consistently with Fama and French (1988), Campbell and Cochrane (1999), Ferson et al. (2003), Pástor and Stambaugh (2009), Binsbergen et al. (2010) and others, we find expected returns to be highly persistent with a monthly persistence coefficient ($\delta_1$) of above 0.75 for all the datasets. The estimated persistence of expected dividend growth rates ($\gamma_1$) instead is generally lower, it ranges between 0.49 and 0.56. Furthermore, shocks to expected dividend growth rates and expected returns are generally positively correlated.

We next examine the identification of speculative bubbles, the prediction of returns and dividend growth rates, and the variance decomposition of the price-dividend ratio and unexpected returns in Sections 6.1, 6.2, and 6.3, respectively.

6.1 Bubble Identification

In this section, we analyze the smoothed surviving-probabilities $P(S_t = 1|Y)$ that the bubble process has been in Regime 1 at time $t$, $(t = 1, \ldots, T)$, in order to distinguish between dates on which either the surviving bubble Regime 1 or the collapsing bubble Regime 2 has been in force. The business-cycle literature generally suggests using the following decision rule: Regime 1 has been in force if $P(S_t = 1|Y) > 0.50$, while Regime 2 has been in force if $P(S_t = 1|Y) \leq 0.50$ (see for instance Goodwin, 1993). However, we expect the bubble process to be in the surviving Regime 1 most of the time, and in the collapsing Regime 2 only for few short periods. This intuition is confirmed by our results which show that the expected duration of the surviving Regime 1 is higher than the expected duration of the collapsing Regime 2, moreover it is confirmed by the findings of Al-Anaswah and Wilfling (2011), and Lammerding et al. (2013). Hence, the threshold value of 0.50 may fail to correctly identify regime switch dates. We follow Al-Anaswah and Wilfling (2011) who suggest to take into consideration the first two moments of $P(S_t = 1|Y)$. They adopt the threshold value $m - 2sd$, where $m$ and $sd$ are respectively the sample mean and sample standard deviation of $\{P(S_t = 1|Y)\}_{t=1,\ldots,T}$. 
Figures 3 and 4 display the time series of the smoothed surviving-probabilities and the log-price-dividend for the artificial datasets, shaded areas denote bubble collapsing regime dates. Figure 3 refers to the artificial datasets 1 and 2 for which our procedure is able to identify eleven out of twelve bubbles. In dataset 3 (upper panel Figure 4) we identify twelve out of fourteen bubbles. The mid panel of Figure 4 refers to dataset 4, in this case all the bubbles are detected correctly, and in dataset 5 (bottom panel Figure 4) we can identify twelve out of thirteen bubbles.
Summing up, our methodology correctly identifies 92.27% of all the bubble collapsing dates in the five artificial datasets. We also observe that our procedure may fail to recognize bubbles of smaller size, in particular we may fail to identify those bubbles which emerge after a bubble of a bigger size. Further, our procedure never signals a bubble which has no counterpart in the price process.

We now turn our attention to the results for the real-world datasets. Figure 5 graphs the smoothed surviving-probabilities and the log-price-dividend ratio for the US. We identify only one collapsing period from October 2008, that is after the collapse of the investment
bank Lehman Brothers, until May 2009. The smoothed surviving-probabilities slightly
decrease in two episodes at the beginning of the series and after the 2008 collapse, however
the decrease is not sharp enough to be signaled as a bubble collapse. Therefore, we do
not detect the internet bubble bursting at the beginning of the new millennium. Similar
comments apply to the United Kingdom (Figure 6) for which the series of the price-
dividend ratio is strongly correlated with that of the US.

Figure 5: Smoothed Surviving-probabilities and Log-price-dividend - United States

Figure 6: Smoothed Surviving-probabilities and Log-price-dividend - United Kingdom
For Malaysia (Figure 7), we observe three clusters of smoothed probabilities indicating collapsing regimes. The first lasts from November 1997 until December 1998, the second from May 1999 until October 1999, and the third is in January 2000 which reflect the Asian financial crisis. By end of 1997, Malaysian ratings fell from investment grade to junk. In January 1998, the Malaysian currency (the ringgit) had already lost 50% of its value to the US dollar. The economy started to recover in 1999.

Japan (Figure 8) was affected less significantly by the Asian financial crises. The smoothed surviving-probabilities signal collapsing regimes in November 1997 and April 1999. Further, like the US and the United Kingdom, Japan experiences a collapse from October to November 2008 in correspondence of the 2008 global financial crises. Figure 9 display the results for Brazil. The smoothed probabilities exhibit some clusters signaling a collapsing regime from November 1997 until April 1998, from July 1998 until September 1998, and from December 1999 until January 2000. These episodes reflect the Brazilian stock-market and currency crisis which culminated in a sharp devaluation of the Brazilian currency (the real) to the US dollar in 1999. The effect was caused by the 1997 Asian financial crisis which led Brazil to increase interest rates and to institute spending cuts and tax increases in an attempt to maintain the value of its currency. The devaluation also precipitated fears that the ongoing economic crisis in Asia would spread to South America, as many South American countries were heavily dependent on industrial exports from Brazil. We register also a drop in the smoothed surviving-probabilities in May 2001 in correspondence of the Brazil energy crises, and in November 2008, one month after the US, the United Kingdom and Japan collapse for the 2008 global financial crises. However, we do not detect a collapsing regime associated with the 2014 Brazilian economic crises.
6.2 Prediction of Returns and Dividend Growth Rates

We now investigate the in-sample predictability of our state space model. We use the estimated series of expected dividend growth rates ($\hat{g}_t$) and expected returns ($\hat{\mu}_t$) as if they were observables and we regress them on realized dividend growth rates and realized returns.

\[
\Delta d_{t+1} = \alpha_d + \beta_d \hat{g}_t + \epsilon_{d,t+1},
\]
\[
r_{t+1} = \alpha_r + \beta_r \hat{\mu}_t + \epsilon_{r,t+1}.
\]

Table 8 shows the results for the artificial datasets, for dividend growth rates the $R^2$ ranges from 74.07% to 78.89% while for returns it is between 4.04% and 20.71%. 

Table 8 shows the results for the artificial datasets, for dividend growth rates the $R^2$ ranges from 74.07% to 78.89% while for returns it is between 4.04% and 20.71%.
Expected dividend growth rates explain a large fraction of actual dividend growth, while the fraction of explained return variability is lower. Also, given that the goodness-of-fit measures of returns vary substantially in the five datasets, we argue that the return predictability features of this model are less robust with respect to the dividend predictability features.

Figures 10, 11 and 12 plot the realized and estimated expected dividend growth rates on the left, and the realized and estimated expected returns on the right for the five artificial datasets. The realized and expected dividend growth series are strongly correlated, however the latter series is more stable. Concerning returns, their expectation is less volatile than the series of realized returns. Further, the correlation between realized and expected returns is lower than that for dividend growth rates.

<table>
<thead>
<tr>
<th>Dataset 1</th>
<th>Dataset 2</th>
<th>Dataset 3</th>
<th>Dataset 4</th>
<th>Dataset 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta d_{t+1}$</td>
<td>$r_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
<td>$r_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0473**</td>
<td>-0.1948**</td>
<td>-0.001</td>
<td>0.1145**</td>
</tr>
<tr>
<td>$\hat{g}_t$</td>
<td>2.0699**</td>
<td>(0.0785)</td>
<td>2.0522**</td>
<td>(0.0763)</td>
</tr>
<tr>
<td>$\hat{\mu}_t$</td>
<td>6.8698**</td>
<td>(1.5246)</td>
<td>6.5605**</td>
<td>(0.9837)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7783</td>
<td>0.093</td>
<td>0.7853</td>
<td>0.1834</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.7772</td>
<td>0.0884</td>
<td>0.7842</td>
<td>0.1793</td>
</tr>
</tbody>
</table>

We report regression results for respectively dividend growth rates and returns on their estimated expected values represented by $\hat{g}_t$ and $\hat{\mu}_t$. Standard errors are reported in parenthesis. Note: **$p \leq 0.05$, *$p \leq 0.1$.

Figure 10: Realized and Expected series - Artificial dataset 1.

The solid lines represent the realized time series of dividend growth rates (left) and returns (right), while the dashed lines represent the estimated series of expected dividend growth rates (left) and expected returns (right).
Figure 11: Realized and Expected series - Artificial datasets 2 to 4. The solid lines represent the realized time series of dividend growth rates (left) and returns (right), while the dashed lines represent the estimated series of expected dividend growth rates (left) and expected returns (right).
Figure 12: Realized and Expected series - Artificial dataset 5.
The solid lines represent the realized time series of dividend growth rates (left) and returns (right), while the dashed lines represent the estimated series of expected dividend growth rates (left) and expected returns (right).

Table 9 reports the results for the real-world datasets. The $R^2$ values for dividend growth rates are quite high in all the datasets, the highest value is recorded for the US where it is equal to 70.49% while the lowest value is registered for Brazil where it is equal to 49.10%.\(^6\) Instead, the coefficient $\beta_r$ in the regressions for returns is significant only for Brazil and the goodness-of-fit measure is equal to 3.12%. In the other countries the $R^2$ values for returns are less than 1% meaning that the estimated series of expected returns ($\hat{\mu}_t$) do not help in predicting stock returns. Visual inspection of Figures 13–17 confirms what we have observed in the regression results.

Table 9: Regression results - Real-world datasets

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>United Kingdom</th>
<th>Malaysia</th>
<th>Japan</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta d_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
<td>$\Delta d_{t+1}$</td>
</tr>
<tr>
<td>$r_{t+1}$</td>
<td>$r_{t+1}$</td>
<td>$r_{t+1}$</td>
<td>$r_{t+1}$</td>
<td>$r_{t+1}$</td>
<td>$r_{t+1}$</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0041**</td>
<td>0.0062**</td>
<td>0.0069**</td>
<td>0.0032</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.0031)</td>
<td>(0.0002)</td>
<td>(0.0033)</td>
<td>(0.0033)</td>
</tr>
<tr>
<td>$\hat{\gamma}_t$</td>
<td>3.8308**</td>
<td>3.9518**</td>
<td>4.9302**</td>
<td>4.4305**</td>
<td>6.4918**</td>
</tr>
<tr>
<td></td>
<td>(0.1606)</td>
<td>(0.1851)</td>
<td>(0.2481)</td>
<td>(0.1896)</td>
<td>(0.4284)</td>
</tr>
<tr>
<td>$\hat{\mu}_t$</td>
<td>0.2511</td>
<td>0.2636</td>
<td>-0.0834</td>
<td>0.0901</td>
<td>0.5914**</td>
</tr>
<tr>
<td></td>
<td>(0.1722)</td>
<td>(0.1784)</td>
<td>(0.3721)</td>
<td>(0.1468)</td>
<td>(0.2136)</td>
</tr>
</tbody>
</table>

We report regression results for respectively dividend growth rates and returns on their estimated expected values represented by $\hat{\gamma}_t$ and $\hat{\mu}_t$. Standard errors are reported in parenthesis. Note:** $p \leq 0.05$, * $p \leq 0.1$.

\(^6\)Since the dividend growth predictability we find may be driven by the seasonality in dividend payments, we apply a stable seasonal filter to estimate the seasonal component in the series of dividend growth rates and expected dividend growth rates. Then we estimate the regression equation using the deseasonalized series. The dividend growth predictability is confirmed and the results are available upon request.
The solid lines represent the realized time series of dividend growth rates (left) and returns (right), while the dashed lines represent the estimated series of expected dividend growth rates (left) and expected returns (right).

Figure 13: Realized and Expected series - United States

Figure 14: Realized and Expected series - United Kingdom

Figure 15: Realized and Expected series - Malaysia
The solid lines represent the realized time series of dividend growth rates (left) and returns (right), while the dashed lines represent the estimated series of expected dividend growth rates (left) and expected returns (right).

6.3 Variance Decomposition

In this section we derive the variance decomposition of both the price-dividend ratio and unexpected returns. The variance decomposition of the price-dividend ratio is given by

\[
\text{Var}(pd_t) = B_2^2\text{Var}(g_t) + B_1^2\text{Var}(\mu_t) + \text{Var}(b_t) - 2B_1B_2\text{Cov}(g_t,\mu_t) - 2B_1\text{Cov}(b_t,\mu_t) + 2B_2\text{Cov}(g_t,b_t).
\]

(33)

The first term, \(B_2^2\text{Var}(g_t)\), represents the variation in the price-dividend ratio due to expected dividend growth rate variation. The second term, \(B_1^2\text{Var}(\mu_t)\), measures the
variation in the price-dividend ratio due to expected return variation. The third term, \( \text{Var}(b_t) \), accounts for the variation in the price-dividend ratio due to bubble variation. The remaining terms represent the covariation among these three components. The unexpected returns can be written as

\[
r_{t+1} - \mu_t = \rho B_2 \epsilon_{t+1}^b - \rho B_1 \epsilon_{t+1}^\mu + \rho \epsilon_{t+1}^b + \epsilon_{t+1}^d.
\] (34)

As before, we decompose the unexpected return variation into the influence of dividend growth variation, discount rate variation, bubble variation and the covariance among these terms.

Table 10 summarizes the results for the variance decomposition of the price dividend ratio and unexpected returns for the real-world datasets. We use sample covariances and we standardize all terms so that the sum is equal to 100%. We find that in the surviving bubble regime, most of the variation in price-dividend ratio is related to the bubble variation. Specifically, bubble variation accounts for more than 50% of the price-dividend variation in all the countries under study with the exception of Brazil where it accounts for about 36%. Instead, consistent with Binsbergen et al. (2010), in the collapsing bubble regime discount rate variation accounts for most of the variation in the price-dividend ratio. Again, consistent with Binsbergen et al. (2010) we document that dividend growth plays a major role in explaining unexpected returns variation in the surviving bubble regime, while discount rate variation accounts most of the variation in unexpected returns in the collapsing bubble regime with the exception for Brazil. Also, bubble variation explains a large share of unexpected return variation in the surviving bubble regime.

Table 10: Variance decomposition - Real-world datasets

<table>
<thead>
<tr>
<th>Regimes</th>
<th>United States</th>
<th>United Kingdom</th>
<th>Malaysia</th>
<th>Japan</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Discount rate</td>
<td>7.5148</td>
<td>96.4404</td>
<td>7.8575</td>
<td>88.1134</td>
<td>45.2324</td>
</tr>
<tr>
<td>Div. growth</td>
<td>0.8453</td>
<td>0.4449</td>
<td>1.0318</td>
<td>0.9170</td>
<td>2.6400</td>
</tr>
<tr>
<td>Bubble</td>
<td>63.5497</td>
<td>0.7822</td>
<td>52.8156</td>
<td>3.2772</td>
<td>56.4466</td>
</tr>
<tr>
<td>Covariances</td>
<td>28.0902</td>
<td>2.3325</td>
<td>38.2952</td>
<td>7.6924</td>
<td>-4.3190</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Panel B: Unexpected returns
| Discount rate | 124.1594      | 100.6654      | 82.0432  | 121.8246 | 80.4089 | 93.1395 |
| Div. growth  | 172.4526      | 0.7604        | 150.4182 | 12.3725 | 268.2840 | 4.7660 |
| Bubble      | 95.0187       | 0.3956        | 92.3121  | 1.2326 | 103.3152 | 1.9123 |
| Covariances | -291.631      | -1.821        | -224.773 | -35.430 | -352.008 | 0.1820 |

We report the decomposition of the price-dividend variation and the unexpected returns variation into discount rate variation, dividend growth variation, bubble variation and the covariances among these three terms.
7 Conclusions

We have shown that in the surviving bubble regime, most of the variation in the price-dividend ratio is related to the bubble variation. Specifically, bubble variation accounts for more than 50% of the price-dividend variation in all the countries under study with the exception of Brazil where it accounts for about 36%. Further, bubble variation explains also a large share of unexpected return variation in the surviving bubble regime.

Hence, we argue that present-value models should not ignore the bubble component of stock prices. The paper proposes to incorporate a speculative bubble subject to a surviving and a collapsing regime into the present-value model by Binsbergen et al. (2010), who adopts a latent variables approach to estimate expected returns and expected dividend growth rates. Further, we suggest an econometrically robust Bayesian Markov-Chain-Monte-Carlo (MCMC) methodology to estimate our model.

This study applies our two-regime Markov-switching state space model to artificial as well as real-world datasets. The artificial bubble processes are defined in the sense of Evans (1991), we show that our bubble-detection methodology is able to correctly identify 92.27% of all the bubble collapsing dates in the five artificial datasets, moreover it never signals a bubble which has no counterpart in the price process. These results represent an improvement with respect to the approach discussed in Al-Anaswah and Wilfling (2011). The real-world datasets consist of the monthly time series data for the dividend yield and the price index for the United States, United Kingdom, Malaysia, Japan and Brazil. We find that our framework is able to identify most of the bubble periods classified as such by Kindleberger and Aliber (2003).

In line with Al-Anaswah and Wilfling (2011) and Lammerding et al. (2013), we document the existence of statistically significant Markov-switching in the data-generating process of real-world stock price bubbles. Furthermore, our methodology is also able to predict dividend growth rates as well as returns with $R^2$ values ranging from 74.07% to 78.89% for dividend growth rates and 4.04% and 20.71% for returns in the artificial datasets. In the real-world datasets, we find that dividend growth rates are predictable with $R^2$ values ranging from 70.49% for the US to 49.10% for Brazil. However, the $R^2$ values for returns are less than 1% with the exception of Brazil where it is above 3%.

In sum, our setup allows to model jointly expected dividend growth rates, expected returns and the bubble component of stock prices. As such it may improve conventional methods for the detection of real-time stock-price bubbles allowing an early detection of future bubbles. Moreover, this methodology allows for hypothesis testing of some features of expected dividend growth rates and expected returns such as their persistence.
References


